



# HOW TO DETERMINE AND REMOVE SYSTEMATIC CHANNEL ERRORS USING 3D LINEAR ANGLE SENSORS

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## INTRODUCTION

This application note intends to describe the channel level errors and their impact on reported angle in 3D linear angle sensors when using the device in an on-axis rotary application. This document will also describe how to compensate for these errors using internal customer settings. For completeness in calculations, this document assumes the use of an A31315 device in the XY 1,000 gauss version, but the principles apply to all of Allegro's 3D linear angle sensors. This includes all orientations and field ranges. As such, this document is meant to further customer understanding of how Allegro's family of 3D angle sensors turn channel measurements into angle outputs. It also seeks to show why angle error profiles manifest in the way they do. If further assistance or clarification is required, contact Allegro for support.

## DESCRIPTION OF CHANNEL LEVEL ERROR SOURCES

Allegro's 3D angle sensor ICs provide angle output readings calculated from a selected two out of three axes. Mitigating the error at the channel level improves accuracy with less required reads than performing linearization after angle calculation. Additionally, many Allegro device channel-level correction registers provide temperature compensation. This allows for increased accuracy over the full operating temperature range.

## ERROR SOURCES

Due to part-to-part variation and magnetic variations, channel readings will contain error. There are three main error sources present in the device channels. These error sources include phase, sensitivity, and offset. Noise is not considered as an error source in this calculation.

For all equations in this document, the following is assumed:

$B_{in}$  = Magnitude of the magnetic flux density vector in gauss,

$\theta$  = Angle of the magnetic flux density vector relative to the selected axes in degrees, and

$SENS_{Target}$  = The target factory-trimmed sensitivity in LSB/G.

*NOTE: All other parameters are given in the same units as they appear in the A31315 datasheet.*

The equation below shows the ideal device channel outputs with no error.

Equation 1: Ideal Channel Outputs

$$mag\_out\_a_{ideal} = B_{in} \times SENS_{Target} \times \cos(\theta)$$

$$mag\_out\_b_{ideal} = B_{in} \times SENS_{Target} \times \sin(\theta)$$

The error sources can also be modeled in this manner and are included in the equations below.

Equation 2: Actual Channel Outputs Including Error Sources

$$mag\_out\_a_{Actual} = SENS_{Target} \times \left( 1 + \frac{ERR_{Sens(A)}}{100} \right) \times (B_{in} \times \cos(\theta + ERR_{Pha(A)}) + ERR_{Off(A)})$$

$$mag\_out\_b_{Actual} = SENS_{Target} \times \left( 1 + \frac{ERR_{Sens(B)}}{100} \right) \times (B_{in} \times \sin(\theta + ERR_{Pha(B)}) + ERR_{Off(B)})$$

## DETERMINING CHANNEL ERROR

In order to correct channel-level errors, it is first required to determine how much of each error component is present. The first step is to record the channel data over applied magnet angle so that Equation 2 can be recreated with application-specific data. Since these equations are sinusoidal versus applied angle, a fast Fourier transform (FFT) is the simplest method to convert discrete samples into the equations of interest.

In order to perform the FFT, at least three values are required for each channel. These locations must be evenly spaced about the 0-360 rotation in order to properly reconstruct the equation. It is not required to use any specific starting angle, but for simplicity, it is assumed here that the following angles are used: 0/360, 120, and 240. These angles are input-referred, meaning they are the angles applied to the device not reported by the device, and as such are not affected by angle correction level parameters (i.e., angle gain, linearization, etc.).

*Note: Additional points can be collected, but it is important that they remain equally spaced and that the 0/360 point is not recorded twice, or the Fourier transform will not be valid.*

Channel data should be collected prior to any customer trim registers; this ensures there is no error in calculations due to existing customer trim values. In the A31315, these registers are mag\_out\_a and mag\_out\_b for channels A and B respectively. All calculations refer to channel data collected at these locations.

Equation 3: Definition of Recorded Channel Points

$A_k$  = Recorded points of channel A.

$B_k$  = Recorded points of channel B.

With these data points, an FFT can be performed to change the measured points into the frequency domain. In this instance, the fundamental frequency of the domain is one full rotation of the applied angle. The output of an FFT can be expressed in several different formats; to simplify the calculations later, the FFT is expressed in rectangular format. The generic formula for an FFT is shown below for each channel as an example.

Equation 4: General FFT Format for both Channels

$$fft(A_k) = \sum_{n=0}^{N-1} Re_{A(n)} + i \times Im_{A(n)}$$

$$fft(B_k) = \sum_{n=0}^{N-1} Re_{B(n)} + i \times Im_{B(n)}$$

where:

$N$  = Number of recorded samples,

$Re_{A(n)}$  = The real portion of the output series for point n, and

$Im_{A(n)}$  = The imaginary portion of the output series for point n.

*Note: Equations expressed in terms of the channel A components that do not provide equivalent channel B versions are valid for both channels. Simply substitute B in place of A for all instances in that equation.*

The following equation shows a reconstruction of a channel given in terms of the real and imaginary components of the FFT above.

Equation 5: Reconstructed Front-End Channels Based on Measured Data Points

$$mag\_out\_a_{Measured} = \sqrt{Re_{A(1)}^2 + Im_{A(1)}^2} \times \cos\left(\theta + atan2\left(\frac{Im_{A(1)}}{Re_{A(1)}}\right)\right) + Re_{A(0)}$$

$$mag\_out\_b_{Measured} = \sqrt{Re_{B(1)}^2 + Im_{B(1)}^2} \times \sin\left(\theta + 90 + atan2\left(\frac{Im_{B(1)}}{Re_{B(1)}}\right)\right) + Re_{B(0)}$$

Decompose the equation into the parameters of the sinusoidal waves for future reference:

Equation 6: Key Channel Parameters

$$Mag_A = \sqrt{Re_{A(1)}^2 + Im_{A(1)}^2}$$

$$Phase_A = atan2\left(\frac{Im_{A(1)}}{Re_{A(1)}}\right)$$

$$Offset_{LSB\_A} = Re_{A(0)}$$

## Orthogonality Trimming

The A31315 trims channel phase by modeling it as a cross sensitivity between the two chosen channels. For this reason, only phase mismatch (orthogonality) can be trimmed. This also means that the act of trimming the orthogonality will also have an impact on the channel's sensitivity. Therefore, this trim is calculated and applied first in the device. The mathematical equivalent of the orthogonality trim is shown below:

Equation 7: Channel Registers after Phase Correction Block

$$\text{Channel\_a} = \text{mag\_out\_a}$$

$$\text{Channel\_b} = \text{mag\_out\_b} + \text{phase\_corr} \times \text{mag\_out\_a}$$

When the channel readings are expressed as the function of applied angle, it becomes more apparent how the phase shift is introduced:

Equation 8: Channel Registers in Terms of Applied Angle

$$\text{Channel\_a} = \text{Mag}_A \times \cos(\theta + \text{phase}_A) + \text{Offset}_{LSB\_A}$$

$$\text{Channel\_b} = \text{Mag}_B \times \sin(\theta + \text{phase}_B) + \text{phase}_{corr} \times \text{Mag}_A \times \cos(\theta + \text{phase}_A) + K$$

where:

$$K = \text{Offset}_{LSB\_B} + \text{phase}_{corr} \times \text{Offset}_{LSB\_A}$$

Reducing the channel B formula:

Equation 9: Channel B Output after Phase Correction

$$\text{Channel\_b} = \sqrt{\text{Mag}_B^2 + (\text{phase}_{corr} \times \text{Mag}_A)^2} \times \sin\left(\theta + \text{phase}_B + atan\left(\frac{(\text{phase}_{corr} \times \text{Mag}_A)}{\text{Mag}_B}\right)\right) + K$$

*Note: Actual phase correction is slightly different as the two channels are not 90 degrees out of phase; however, for small orthogonality errors, this approximation holds.*

Since phase correction is only applicable on channel B, it is used to align the two channels to be 90 degrees out of phase with each other. Below shows that equality including the phase correction component.

Equation 10: Definition of Phases Required for Both Channels to be Orthogonal

$$Phase_A = Phase_B + atan\left(\frac{(phase_{corr} \times Mag_A)}{Mag_B}\right) + 90$$

Solving for the phase correction component results in the following:

Equation 11: Phase Correction Required to Satisfy Channel Orthogonality

$$phase_{corr} = \tan(Phase_A - (Phase_B + 90)) \times \frac{Mag_B}{Mag_A}$$

The A31315 contains phase correction with a range of  $\pm 0.125$  cross-sensitivity coefficient, and a step size of  $2^{-11}$  per code.

Equation 12: Phase Correction Code for the A31315

$$phase\_f = \frac{phase_{corr}}{4.88 \times 10^{-4}}$$

XY versions of the A31315 are factory-trimmed for orthogonality. The existing orthogonality trim code is required to be known if attempting to retrim this parameter. All references to orthogonality trim are referred to as changes in trim from the existing code. For example, if three codes of trim are calculated, the existing value should be increased by three codes.

*Note: Since phase correction is not linear, but follows a tangent function, this is not fully accurate. However, for small values around 0, the tangent function appears linear, so it is possible to use this as a linear function in terms of adding customer trim to existing trim codes.*

## Sensitivity Trimming

Since orthogonality trimming influences the magnitude of channel B after the correction is applied, it is necessary to redefine the magnitude of channel B before continuing with sensitivity correction. It is important to note that in the A31315, only one phase trim exists, so for XY versions even if no changes were made to the code it is still necessary to perform this calculation based on the code already stored in EEPROM.

Equation 13: Redefine Channel B Magnitude Based on phase\_f

$$Mag_{B(New)} = \sqrt{Mag_B^2 + (phase\_f \times 4.8828 \times 10^{-4} \times Mag_A)^2}$$

Determining the actual sensitivity once the magnitude is known is simple. To match with the Allegro definition of sensitivity (LSB16/G), simply divide magnitude by the field applied.

Equation 14: Calculate Sensitivity in LSB16 / G

$$SENS_{Measured(A)} = \frac{Mag_A}{B_{in}}$$

In many cases, the desired sensitivity in the end application is not equal to the factory-programmed target sensitivity. Keeping that in mind, the sensitivity correction calculation below requires the user to define a custom target sensitivity ( $SENS_{Custom}$ ) in LSB/G. This should be wide enough to accommodate all magnetic variances in applied fields without saturating the device. If it is desired to use the factory-programmed sensitivity target ( $SENS_{Target}$ ), the A31315 1,000 G version's target sensitivity of 32.125 LSB/G can be used.

Equation 15: Calculating the Sensitivity Correction Required to Obtain the Target Sensitivity for Each Channel

$$SENS_{Correction(A)} = \frac{B_{in} \times SENS_{Custom}}{Mag_A}$$

Converting this value to a programmable code will be based on the device's trim resolution. Using the A31315 as an example, the resolution provided is  $2^{-11}$ , with a range of 0 to 7.9995.

Equation 16: Sensitivity Correction Code for the A31315

$$sens\_c\_a = \frac{SENS_{Correction(A)}}{4.88 \times 10^{-4}}$$

## Offset Trimming

As with sensitivity, since phase correction modified the offset of Channel B, the offset equation must be modified to account for the new offset content. Even if the value was not changed, the existing phase value must be considered.

Equation 17: New Definition of Channel B Offset After Phase Correction

$$Offset_{LSB,B(New)} = Offset_{LSB,B} + phase\_f \times 4.8828 \times 10^{-4} \times Offset_{LSB,A}$$

The A31315 provides offset trim in the range of -32768 to 32764 LSBs, with a resolution of 4 LSB per code. To determine the exact value to program into the device, the following equation should be used:

Equation 18: Offset Correction Code for the A31315

$$ofs\_c\_a = -\frac{Offset_{LSB,A}}{4}$$

## ERROR IMPACT

Each error source has a slightly different impact on angle accuracy. To fully understand them, each one will be analyzed independently.

### Offset Error

Each channel ideally contains no inherent offset or DC component; as a result, offset error is typically referred to as just offset, as any offset value is erroneous. Offset error of each channel impacts angle error as a ratio of the offset to the magnitude of the channel.

The impact of offset is mathematically expressed for the channels below. Both channel magnitude and offset must be represented in the same unit; by default, Allegro uses gauss for measurements of magnetic field.

Equation 19: Determine Angle Error in Radians Due to Channel Offset

$$Angle_{Err\_off(Rad)} = \frac{Offset_{LSB,B}}{Mag_{Channel(B)}} \times \cos(\theta) - \frac{Offset_{LSB,A}}{Mag_{Channel(A)}} \times \sin(\theta)$$

Equation 20: Determine Angle Error in Degrees Due to Channel Offset

$$Angle_{Err\_off(Deg)} = \frac{180}{\pi} \times Angle_{Err\_off(Rad)}$$

## Sensitivity Error

Each channel has a factory-trimmed sensitivity target. However, there will be some error in each channel's sensitivity measurement. The sensitivity error of the channel has no direct impact on angle accuracy. It is the mismatch of sensitivity between the two channels that determines the impact on angle accuracy. This is because sensitivity error that is equal on both channels is no different from applying a magnetic vector with a different amplitude.

The impact of sensitivity mismatch between the two channels is mathematically expressed below:

Equation 21: Calculate Sensitivity Mismatch from Channel Magnitudes

$$Sensitivity_{Mismatch} = \frac{Sensitivity_{Measured(B)} - Sensitivity_{Channel(A)}}{(Sensitivity_{Measured(B)} + Sensitivity_{Measured(A)}) / 2} \times 100$$

Equation 22: Calculate Angle Error Due to Channel Sensitivity Mismatch

$$Angle_{Err\_sens(Deg)} = Sensitivity\_Mismatch \times 0.2866 \times \sin(2\theta)$$

*Note: Equation 22 is a linear approximation of the error function. The actual error is non-linear, but this approximation holds for small scale sensitivity mismatch, such as those seen on trimmed devices.*

## Phase Error

When the actual axis of device sensitivity does not align with the stated axis of sensitivity, a phase error is introduced into the channel readings. This is the equivalent of an angle misalignment when looking at each channel. However, an angle error is introduced when the two channels see a different phase value. This causes the equivalent error of one of the sensors being tilted.

The impact of this phase difference on angle error is shown below:

Equation 23: Calculate the Common Phase Error between the Two Channels

$$Phase_{Common} = \frac{Phase_{ErrB} + Phase_{ErrA}}{2}$$

Equation 24: Calculate the Differential Phase Error between the Two Channels

$$Phase_{Differential} = Phase_{ErrB} - Phase_{ErrA}$$

Equation 25: Determine the Angle Error Caused by Phase Errors

$$Angle_{Err\_phase(Deg)} = Phase_{Differential} \times \cos(2\theta) + Phase_{Common}$$

## Summing All Error Sources

Each of the previously calculated angle error values are independent of each other. As a result, each error component for a given IC can be added together to determine the total angle error profile of the device. Summing together multiple error values calculated based on worst-case datasheet specifications for given channel parameters may lead to overly conservative estimates of total angle error due to the likelihood of a single IC displaying worst-case performance for multiple channel parameters. Using a Monte Carlo simulation will provide a more accurate estimate of actual angle performance than simply stacking worst-case errors.

Equation 26: Summation of all Angle Error Components

$$Angle_{Err\_Total(Deg)} = Angle_{Err\_off(Deg)} + Angle_{Err\_sens(Deg)} + Angle_{Err\_phase(Deg)}$$

*Revision History*

Number	Date	Description	Responsibility
-	December 9, 2020	Initial release	Matthew Mack

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